PHYSICAL REVIEW E VOLUME 55, NUMBER 5 MAY 1997

Beam-channeled laser-wakefield accelerator

G. Shvets and N. J. Fisch
Princeton Plasma Physics Laboratory, Princeton University, Princeton, New Jersey 08543

(Received 23 September 1996)

A hybrid of laser-wakefield and plasma-wakefield accelerators is proposed, in which an intense laser pulse is optically guided by a plasma channel created by the leading portion of a high-current, low-energy electron beam. This accelerator configuration appears to combine advantageously features of both laser-wakefield and plasma-wakefield accelerators. [S1063-651X(97)02505-1]

PACS number(s): 41.75.Lx, 52.75.Di

Because of the very high electric field it can sustain, plasma has been proposed [1–7] as a medium for high-gradient particle acceleration. Moreover, in a plasma it is possible to convert strong transverse fields, characteristic of lasers and fast-moving charged particles, into longitudinal waves, which are suitable for accelerating particles to high energy. The plasma waves can be excited by relativistic electron bunches [2–4] in a plasma-wakefield accelerator or by intense laser pulses [1,8–11] in a laser-wakefield accelerator.

The plasma-wakefield accelerator promises fields in excess of 1 GeV/m, with a long interaction distance due to self-focusing of the driving bunch. Unfortunately, the fundamental wake theorem [2,12] limits the transformer ratio (the peak accelerating field E_+ experienced by the accelerated electrons over the average decelerating field $\langle E_- \rangle$ acting on the driving electron bunch) to less than 2 for symmetric bunches. Thus, accelerating electrons to 1 GeV energy would require a 0.5-GeV driving beam.

The transformer ratio might be increased to $T \approx \pi N_p$ by shaped bunches [3,4], with a slow linear rise in density over N_p plasma periods and an abrupt termination at $\xi = N_p \lambda_p$, where $\lambda_p = 2\pi c/\omega_p$ and $\omega_p = \sqrt{4\pi e^2 n_0/m}$ are the plasma wavelength and plasma frequency, respectively. However, an abrupt termination over a distance smaller than a collisionless skin depth may be difficult to accomplish in practice.

On the other hand, laser acceleration is limited by diffraction of the laser pulse, which reduces the propagation distance to about a Rayleigh length $Z_R = \pi r_L^2/\lambda_0$, where λ_0 is the laser wavelength. Laser pulses might be guided by plasma channels [10,11], which have been formed [13] by a hydrodynamic expansion of a laser breakdown spark in an ambient gas. Differential ''leakage'' rates [13] for different quasibound transverse laser modes allows for some control over the modal content and Raman-type instabilities of the laser pulse [14].

We suggest a method of creating a narrow plasma channel that utilizes the leading portion of a high-current electron beam. An intense laser pulse, timed with the electron bunch so that it is placed several plasma wavelengths behind the head of the pulse, is guided by the plasma channel, generated by the beam, and excites a strong plasma wake. This wake accelerates the portion of the electron beam that trails behind the laser pulse. Since the width and density depression of the plasma channel are determined by the electron beam, they can be accurately controlled by the beam optics. Creating the channel by the same beam, a slice of which is accelerated, aligns the injected electrons and the plasma channel. We demonstrate that a very compact GeV accelerator can be produced using this technique. While it is still premature to speculate on whether such an accelerator can become a single stage of a multi-GeV collider, it will likely find many other useful applications wherever ultrashort high-energy bunches are required.

It is essential that the leading portion of the electron beam, which creates the plasma channel, is several plasma periods long, and that the beam density is smaller than the plasma density. This results in a high transformer ratio, with the decelerating field experienced by the leading beam much smaller than the accelerating field experienced by the trailing portion of the beam. The transformer ratio for the present accelerator scheme does not have the meaning of transferring energy from the driving beam to the trailing beam (or the wake left behind), since almost all of the energy for acceleration comes from the laser pulse.

It appears that a small portion of the beam can then be accelerated to GeV energies if (i) the electron beam is stable and guides the laser pulse, (ii) the plasma density provides sufficient focusing to the beam and a sufficient accelerating gradient; or (iii) the beam emittance is practically attainable, consistent with reasonable laser power required for acceleration. We find that all these requirements might be met in, for example, the compact accelerator, with the parameters given in Table I.

In the following, we analyze in detail the requirements (i)–(iii). We show that the parameters that simultaneously meet these requirements also lead to rather interesting behavior of the electron beam, which is subjected to the self-consistent wakefields generated by both the laser and the beam itself.

(i) Assume a flat-top axisymmetric electron beam radial density profile of radius r_b, much longer than a plasma period,

$$n_b(\xi,r) = n_{b0} H(r_b - r) \exp - \xi^2/(2\sigma_z^2), \tag{1}$$

with $\sigma_z \gg c/\omega_p$, so that the density depression in the plasma closely follows the beam density n_b and the inductive electric field, which slows down the beam, is very small. Injecting a laser pulse near $\xi = 0$ insures that the laser pulse experiences the largest density depression n_{b0}. Since the relatively light plasma electrons are replaced by highly rela-

1063-651X/97/55(5)/6297(4)/$10.00 55 6297 © 1997 The American Physical Society

TABLE I. List of parameters for beam-channeled laser wakefield accelerator.

E-beam energy	γmc^2	25 MeV
E-beam current	I_b	4.25 kAmp
E-beam radius	r_b	17 μ
E-beam emittance	ϵ_n	30 πmm mrad
E-beam density	n_b	10^{17} cm^{-3}
Bunch length	σ_z	1.0 psec
Laser radius	r_L	34μ
Vector potential	a_0	0.9
Laser intensity	I	2.3×10^{18} W/cm^2
Laser power	P	80 TW
Pulse duration	τ_L	85 fs
Plasma density	n_p	4×10^{17} cm^{-3}
Laser gamma factor	γ_g	50.0
Accelerating gradient	W_z	16.3 GeV/m
Final energy	$\gamma_f mc^2$	1.0 GeV
Total length	L_t	6.3 cm
Rayleigh length	Z_r	0.3 cm

tivistic beam electrons, the refractive index becomes $n_{\rm ref}=1-\omega_p^2(r)/\omega_0^2$. It can be demonstrated [18] that the wavelength-independent product $V=k_{pb}r_b=\sqrt{4I/I_A}$, where $k_{pb}^2=4\pi n_{b0}e^2/mc^2$ and $I_A=17$ kA is Alfvén current, plays the role of a fiber parameter for the laser pulse. To confine one mode only, take $V<1.8$. The appearance of a second mode could lead to unwanted hosing instabilities [15,16]. Numerical studies of leaky channels, which have a significantly higher attenuation coefficient for higher-order modes than for the fundamental mode [14], demonstrated that various Raman instabilities (such as hosing [15,16]) are strongly suppressed. On the other hand, choosing the fiber parameter to be too small leads to excessive transverse spreading and unwanted loss of laser power. Thus we assume fiber parameter to be unity. For $k_{pb}r_b=1$ we find that the amplitude of the laser pulse drops to half of its amplitude at $r\approx2r_b$ [11]. The effective spot size of the laser pulse is also about $r_L=2r_b$. Peak current of $I_b=I_A/4\approx4.25$ kA is thus required for effective guiding of the laser pulse.

(ii) Propagating a dense electron beam through plasma results in the return current [17]. In order that the beam self-focusing not be strongly reduced by the return current in the plasma, choose the ambient plasma density to be small enough that the beam radius does not exceed the collisionless skin depth c/ω_p by much. Nonrelativistically, we estimate the return current density on axis as $J_-=F_R(r=0)en_bc$, which reduces the on-axis magnetic field pinching force by a factor $\eta=1-F_R(r=0)$, where the reduction factor F_R is associated with the finite radius of the electron bunch and is given by [21]

$$F_R(r)=\begin{cases}1-k_pr_bK_1(k_pr_b)I_0(k_pr) & \text{for } r<r_b\\ k_pr_bI_1(k_pr_b)K_0(k_pr) & \text{for } r>r_b.\end{cases} \qquad (2)$$

For $k_pr_b=2$, the reduction on axis is then $F_R(r=0)\equiv F_R=0.72$.

For a self-focused beam, the normalized emittance, beam radius, and beam energy are related through [18]

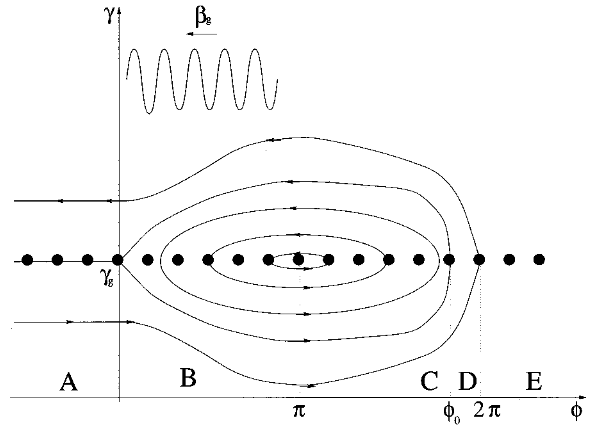

FIG. 1. Schematic of particle phase space in a beam-channeled laser wakefield accelerator. *A*: guiding beam; *B*: laser pulse location, region of beam erosion; *C*: trapped accelerated electrons; *D*: untrapped accelerated electrons; *E*: trailing beam. Initially monoenergetic electrons are schematically shown as shaded circles.

$$\epsilon_n=r_b\left(\frac{\gamma\eta I_b}{2I_A}\right)^{1/2}. \qquad (3)$$

(iii) Choose the electron beam spot size so that (1), the radius is not so large as to make the required laser power exceed the capabilities of the present and soon-to-come laser systems; and (2) the radius is not so small as to make the required beam emittance too stringent. The laser power and emittance given in Table I are achievable with today's technologies [19]. In fact, one of the RF photocathode rf guns currently under development [20] delivers almost exactly the required beam parameters.

To study the excitation of the accelerating wake, we consider a circularly polarized, nonevolving laser pulse, with a flat-top longitudinal profile of width equal to half plasma wavelength, and with

$$a_0^2(\xi,r)=a_0^2\ [H(\xi)-H(L-\xi)]\ \psi(r), \qquad (4)$$

where $\psi(r)$ is a normalized transverse profile of a confined laser mode, derived in Ref. [11], $a_0=eA_0/mc^2$ is a normalized vector potential, and $L=\lambda_p/2$. In the linear regime $(n_b\ll n_0,\ a_0^2\ll1)$, for $\gamma_g\gg1$, the longitudinal field on axis is given by

$$E_z(\xi,r=0)=4\pi e\int_{-\infty}^{\xi}d\xi'\,\frac{\sin k_p(\xi-\xi')}{k_p}\times\left[n_0\frac{\partial}{\partial\xi'}\left(\frac{a_0^2}{2}\right)+F_R(r=0)\frac{\partial n_b(\xi')}{\partial\xi'}\right], \qquad (5)$$

where $\xi=ct-z/\beta_g$ is a coordinate comoving with a bunch, n_b is density of the electron bunch, and $a=eA/mc^2$ is a normalized vector potential of a circularly polarized laser. These equations apply also for linear polarization if a_0^2 is replaced by $a_0^2/2$.

Figure 1 schematically presents the phase space of beam electrons, which are assumed to have the initial velocity equal to the group velocity of the laser $c\beta_g$. Electrons in region A are, by causality, unaffected by the laser and are only subject to a weak, self-generated decelerating field. Their function is to create a plasma channel and to guide the laser pulse. On the other hand, electrons in region B experience a strong decelerating wake, rapidly slip forward in phase, and are transferred into the accelerating region C. For GeV-scale acceleration, our attention may be confined to the wake generation in region C, avoiding the details of the dynamics of the beam electrons and the wake for $\xi > \lambda_p$ (region E).

Making use of disparate time scales for particle deceleration and acceleration, the wake generation process can be separated into two stages: the initial stage, when erosion of the decelerated portion of the electron beam takes place, and the final stage, when beam electrons are accelerated to high energy in a self-consistent wakefield, generated by both the laser and the beam. During the erosion stage, the beam electrons initially residing in region B slow down and slip into region C. Depending on laser intensity, some or all of the initially decelerated electrons, injected with velocity equal to phase velocity of the wake are trapped. Only the case of high laser intensity is schematically shown in Fig. 1, indicating that all beam electrons between $\phi=0$ and $\phi=2\pi$ are trapped. In both cases, electrons, which are initially at a decelerating phase, rapidly execute half a synchrotron oscillation, thus arriving at an accelerating phase. Their departure generates a sharp edge in the electron density, which modifies the wake. Untrapped electrons (if any) slip further in ξ, leaving the region of interest, while the trapped electrons are accelerated to high energy. In this paper we calculate a plasma wake, based on a simplifying assumption that beam density is piecewise constant and vanishes in the regions of $E_z > 0$.

The relativistic electron dynamics in a fixed wakefield can be described by a Hamiltonian [21]

$$H = \sqrt{\gamma^2 - 1} - (\gamma/\beta_g) + V(\phi), \tag{6}$$

where $\phi = k_p \xi$ and $E_z = (mc\omega_p/e)\partial V/\partial\phi$. The equations of motion are then given by $\phi' = \partial H/\partial\gamma$ and $\gamma' = -\partial H/\partial\phi$, where the prime denotes differentiation with respect to $z = k_p z$. The wake potential $V(\phi)$ is determined self-consistently from Eq. (5). Depending on the laser intensity, two different regimes are possible: $V(2\pi) < V(0)$ (the high-intensity regime, schematically shown in Fig. 1, $a_0^2 > 4F_R n_{b0}/n_0$) and $V(2\pi) > V(0)$ (low-intensity regime, $2F_R n_{b0}/n_0 < a_0^2 < 4F_R n_{b0}/n_0$). We consider these two regimes separately.

In the high-intensity regime, electrons are trapped in the region $0 < \phi < \phi_0$, where $V(0) = V(\phi_0)$. Electrons from region B are rapidly transferred to region C, after a half-synchrotron oscillation, generating sharp density gradients at $\phi = 0$ and $\phi = \phi_0$. Both the trapped ($\pi < \phi < \phi_0$) and untrapped ($\phi_0 < \phi < 2\pi$) accelerated electrons slip back in ϕ at a much slower rate and can be assumed to be fixed in phase.

Assuming that decelerated electrons from region B are uniformly spread out through region C, we estimate the electron density in region C as $n_b' = n_{b0}\phi_0/(\phi_0 - \pi)$ and the electron density in region D as unchanged at n_{b0}. Introducing $u = (a_0^2/2 - n_{b0}/n_0 F_R)$ and $g = [a_0^2 - (n_{b0}/n_0 + n_b'/n_0)F_R]/u$, the wake potential is then given by

$$V(\phi) = \begin{cases} -u\cos\phi & \text{for } 0 < \phi < \pi, \quad (7) \\ -u(g\cos\phi + g - 1) & \text{for } \pi < \phi < \phi_0, \quad (8) \end{cases}$$

where $1 < g < 2$. The resulting beam density n_b', consistent with such a wake, becomes an implicit function of the laser intensity (characterized by the quantity u) and can be expressed in terms of u and g. The quantity g, itself an implicit function of u, can now be determined from a nonlinear equation:

$$\sin^2\left(\frac{\pi}{2}\,\frac{F_R n_{b0}/n_0}{u(2-g)}\right) = \frac{1}{g}, \tag{9}$$

where $C = F_R n_{b0}/(n_0 u) < 1$.

In the low-intensity regime, the self-consistent wake potential is also described by Eqs. (7) and (8), except that the domain in Eq. (8) is extended from ϕ_0 to 2π. Here, $g < 1$, so that electrons initially at $0 < \phi < \phi_1$ are untrapped, where $V(\phi_1) = V(2\pi)$, and so rapidly leave region $0 < \phi < 2\pi$. At the same time, electrons initially in the region $\phi_1 < \phi < \pi$ are trapped and are transported to the region $\pi < \phi < 2\pi$, resulting in an overall density there of $n_b' = n_{b0}(2\pi - \phi_1)/\pi$. A nonlinear equation for g, analogous to Eq. (9), can be derived for this regime:

$$\sin^2\left(\frac{\pi}{2}\,\frac{u(2-g)}{F_R n_{b0}/n_0}\right) = g. \tag{10}$$

Equations (9) and (10) can be solved numerically for various values of C, corresponding to different laser intensities. The required laser intensity and peak accelerating gradient achieved at $\phi = 3\pi/2$ are given by $a_0^2 = 2n_{b0}/n_0 F_R(1+C)/C$ and $W_z = 4\pi e n_0 g F_R n_b/(k_p C n_0)$.

As an example, consider a high-intensity regime $C = 0.75$ and, using the plasma and beam parameters from Table I, obtain $a_0^2 \approx 0.84$, $\phi_0 \approx 1.82\pi$, and $W_z \approx 16.3$ GeV/m. An important caveat here is that the laser intensity needed to achieve such an accelerating gradient may be somewhat underestimated by the present calculation, which assumes $a_0^2 \ll 1$, which is only marginally satisfied for the suggested parameters.

For the nonevolving potential given by Eq. (8), the equations of motion for beam particles were integrated numerically to verify that the plasma wake, generated by the evolving electron beam, is indeed close to the ones assumed in deriving Eq. (8). In Fig. 2, we compare, at different propagation distances, the theoretically assumed and numerically calculated beam wakefields. Note that the actual beam wakefields become almost undistinguishable from the theoretically assumed wakefields after a propagation distance $z < 0.1 z_{\mathrm{acc}}$, where $z_{\mathrm{acc}} = 8000c/\omega_p \approx 6.3$ cm is the distance required for 1 GeV acceleration. The beam electrons occupying the region around $\xi = 3\lambda_p/2$ are accelerated by being trapped in the plasma wakefield. The maximum energy the trapped electrons can acquire is limited by the height of the separatrix; for $u \ll 1$, this is [21] $\gamma_{\max} = 4\gamma_g^2 g u = (1.3$ GeV$)/mc^2$. Note also that the beam wakefield deviates from the analytic values for $z \approx z_{\mathrm{acc}}$ because of the bounce motion

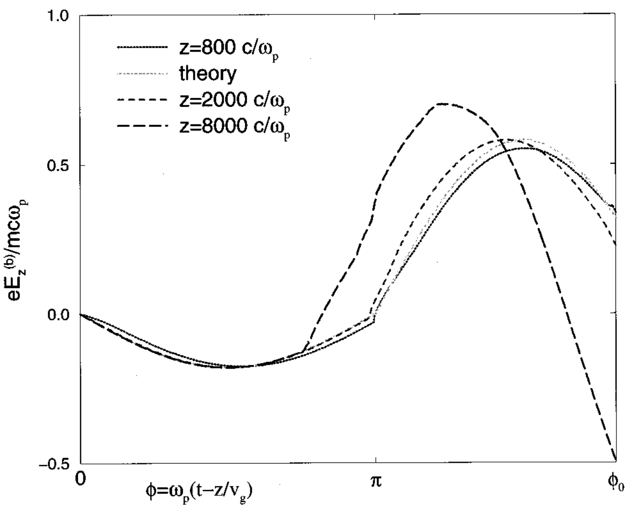

FIG. 2. Wake generated by the bunched beam; $a_0=0.84$, $\phi_0=1.82\pi$. Final energy 1 GeV is reached at $z_{\rm acc}=8000c/\omega_p$.

of beam electrons near the equilibrium point $\phi=\pi$. Self-consistent particle simulations are needed to accurately include the two-dimensional and relativistic effects and to refine the calculation of the beam wake for long acceleration distances. Another important caveat is that it is by no means certain that the electron beam can be propagated over the required distance without degradation caused by electron-hose instability [22,23]. However, substantial plasma return current inside the beam is likely to mitigate the hosing instability by introducing spreads in betatron frequencies of beam electrons and in ambient plasma frequency (through the Doppler shift) [24]. We also note that, even if hosing distorts the overall shape of the beam, the laser may still be guided by the displaced plasma channel.

In summary, we show how an intense (over 4 kA peak current) electron beam can create a plasma channel that guides a short laser pulse, generating a plasma wake for electron acceleration. Among the advantages of this scheme over the conventional laser-wakefield designs is the long acceleration distance (not limited by diffraction); perfect alignment between the channel and the accelerated electrons; absence of potentially dangerous transverse instabilities of the laser pulse such as hosing; and control over the profile of a plasma channel by varying the electron beam profile.

The authors acknowledge insightful discussions with J. S. Wurtele and T. C. Katsouleas. This work was supported in part by the U.S. DOE under Contract No. DE-AC02-76-CHO3073.

[1] T. Tajima and J. M. Dawson, Phys. Rev. Lett. **43**, 267 (1979); L. M. Gorbunov and V. I. Kirsanov, Zh. Exp. Teor. Fiz. **93**, 509 (1987) [Sov. Phys. JETP **66**, 290 (1987)]; C. Joshi, *et al.*, Nature **311**, 525 (1984).

[2] R. D. Ruth, A. Chao, P. L. Morton, and P. B. Wilson, Part. Accel. **17,** 171 (1985).

[3] K. L. F. Bane, P. Chen, P. B. Wilson, IEEE Trans. Nucl. Sci. **32,** 3524 (1985).

[4] T. Katsouleas, Phys. Rev. A **33,** 2056 (1986).

[5] J. S. Wurtele, Phys. Fluids B **5**, 2363 (1993); Phys. Today **47**, (7) 33 (1994), and references therein.

[6] P. Sprangle and E. Esarey, Phys. Fluids B **4**, 2241 (1992).

[7] C. E. Clayton, *et al.*, Phys. Rev. Lett. **30**, 37 (1993).

[8] P. Sprangle, E. Esarey, A. Ting, and G. Joyce, Appl. Phys. Lett. **53,** 2146 (1988).

[9] J. M. Rax and N. J. Fisch, Phys. Fluids B **4**, 1323 (1992); J. M. Rax and N. J. Fisch, Phys. Fluids B **5**, 2680 (1993).

[10] E. Esarey, P. Sprangle, J. Krall, A. Ting, and G. Joyce, Phys. Fluids B **5**, 2578 (1993).

[11] T. C. Chiou, T. Katsouleas, C. Decker, W. B. Mori, J. S. Wurtele, G. Shvets, and J. J. Su, Phys. Plasmas **2,** 310 (1995).

[12] A. W. Chao, in *Physics of High Energy Particle Accelerators,* Proceedings of lectures presented at the Second Annual U.S. Summer School on High Energy Particle Accelerators, edited by M. Month, AIP Conf. Proc. No. 105 (AIP, New York, 1982).

[13] C. G. Durfee and H. M. Milchberg, Phys. Rev. Lett. **71**, 2409 (1993); C. G. Durfee, J. Lynch, and H. M. Milchberg, Phys. Rev. E **51**, 2368 (1995); H. M. Milchberg, T. R. Clark, C. G. Durfee, T. M. Antonsen, and P. Mora, Phys. Plasmas **3,** 2149 (1996).

[14] T. M. Antonsen, Jr. and P. Mora, Phys. Rev. Lett. **744**, 4440 (1995).

[15] G. Shvets and J. S. Wurtele, Phys. Rev. Lett. **73,** 3540 (1994).

[16] P. Sprangle, J. Krall, and E. Esarey, Phys. Rev. Lett. **73,** 3544 (1994).

[17] J. D. Lawson, J. Elect. Control. **3,** 587 (1957); D. A. Hammer *et al.*, Phys. Fluids **13,** 1831 (1970).

[18] D. H. Whittum, Ph.D. thesis, UC Berkeley, 1990.

[19] B. E. Carlsten and S. E. Russel, Phys. Rev. E **53,** R2072 (1996); E. Colby *et al.*, in *Proceedings of the Advanced Accelerator Concepts Workshop, Fontana, Wisconsin,* edited by P. Schoessow, AIP Conf. Proc. No. 335 (AIP, New York, 1995), p. 708.

[20] The Fermilab Test Facility Photoinjector is projected to deliver $Q=10$ nC of charge in $\sigma_b=1$ psec and normalized emittance $\epsilon=40\pi$ mm mrad (P. Colestock, private communication).

[21] E. Esarey, P. Sprangle, J. Krall, and A. Ting, IEEE Trans. Plasma Science **24,** 252 (1996).

[22] D. H. Whittum, W. M. Sharp, S. S. Yu, M. Lampe, and G. Joyce, Phys. Rev. Lett. **67,** 991 (1991).

[23] M. Lampe, G. Joyce, S. P. Slinker, and D. H. Whittum, Phys. Fluids B **5,** 1888 (1993).

[24] D. H. Whittum (unpublished).